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LETTER TO THE EDITOR

The spontaneous Aharonov–Casher effect associated with partially spin-polarized states in one-dimensional mesoscopic rings

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Abstract. The spontaneous Aharonov–Casher (AC) effect associated with partially spin-polarized states in one-dimensional mesoscopic rings is investigated for the first time. The results obtained seem to be novel and interesting. Particularly, for an even number of electrons, the spontaneously introduced AC flux does not depend on the polarization parameter N_p; while for an odd number of electrons, it increases with a specific set of N_p and decreases with the other set. Moreover, the polarization magnetic field is calculated analytically.

Aharonov and Casher have predicted that a neutral particle with a magnetic moment will exhibit a force-free interference effect when its path encloses a charged wire [1, 2]. This Aharonov–Casher (AC) effect is the dual of the Aharonov–Bohm (AB) effect describing the force-free interaction between an electrical charge and the magnetic flux enclosed. Thus, similarly to the AB problem, the electrons possessing spin-$\frac{1}{2}$ in a mesoscopic ring pierced by a charged rod are expected to display persistent spin currents [3], which are periodic in the 'AC flux'. In this case, the persistent spin current itself also produces AC flux in addition to the externally applied flux. Recently, Choi pointed out that the spin current can be self-sustained without an external electric field in mesoscopic rings [4]. However, the crucial condition that all the electrons have the same spin state σ_z, was assumed in his theory. It is well known that the occupation of electrons must obey the Pauli exclusion principle and the energy levels are discretized significantly for one-dimensional (1D) mesoscopic systems, which implies that the energy of a 1D mesoscopic ring in the case of each quantum state being occupied by one electron with the same spin σ_z, is much larger than that in the case of each state being occupied by two electrons with opposite spins. Therefore, it is unreasonable to speculate that the spin of a many-electron system can be spontaneously and fully polarized without an external polarization field. In this letter, using an approach analogous to treating the flux motion issue, we study the spontaneous AC effect in 1D mesoscopic rings, where the spin of electrons could be partially polarized by introducing a local magnetic field. The results presented seem to be novel and interesting. Particularly, for an even number of electrons, the spontaneously introduced AC flux does not depend on the polarization parameter $N_p = \sum \sigma_z$; but for an odd number of electrons, it increases with a specific set of N_p and decreases with the other set.

First of all, we would like to emphasize that we here concentrate our attention only on the pure AC effect, namely, we neglect the interaction between electrons and regard electrons as neutral particles with spin-$\frac{1}{2}$. In this case, there is no AB effect in play. In the presence of AC flux, the mechanical momentum of an electron can be written as

$p - \mathcal{E} \times \mu/c = p - \sigma_z \mu_B \mathcal{E} \times \hat{z}/c$, where $\sigma_z (= \pm 1)$ represents the spin state, μ_B and $\mathcal{E}$ are respectively the Bohr magneton and the electric field [1, 2]. It has been demonstrated that the interaction term $\mathcal{E} \times \mu/c$, although appearing as a local interaction, represents a non-local interaction, and the AC effect is essentially non-local in its nature [5]. Therefore, we are able to work with a gauge in which the field does not appear explicitly in the Hamiltonian and the current operators, but enters the calculation via the flux-modified boundary condition

$$\psi(L) = \exp(\mathrm{i}2\pi f_{AC})\psi(0) \tag{1}$$

where $f_{AC} \equiv \Phi_{AC}/\Phi_0$ is the AC flux with

$$\Phi_{AC} = \mu_B/e \oint \hat{z} \cdot (\mathrm{d}l \times \mathcal{E}) \qquad \Phi_0 \equiv hc/e$$

and L is the circumference of the ring with radius R, vanishingly small width a and thickness l. Solving the one-dimensional Schrödinger equation:

$$-\frac{\hbar^2}{2m_e}\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} = E\psi(x)$$

subject to the boundary condition equation (1), one can have the eigenvalue $E_{n\sigma_z}$ and the spin current

$$I_{n\sigma} = \frac{1}{4\pi}\frac{\partial E_{n\sigma}}{\partial f_{AC}}$$

of the $n\sigma$th eigenstate

$$E_{n\sigma_z} = \frac{\hbar^2}{2m_e R^2}(n + \sigma_z f_{AC})^2$$

$$I_{n\sigma_z} = \frac{\hbar^2}{4\pi m_e R^2}(f_{AC} + n\sigma_z)$$

where the energy level index $n = 0, \pm 1, \pm 2, \ldots$. The energy of particles and the spin current in the ring equal to the sum over the individual contribution from each occupied state i.e.

$$E = \sum_{n\sigma_z} E_{n\sigma_z} \tag{2}$$

$$I_s = \sum_{n\sigma_z} I_{n\sigma_z} = \frac{1}{4\pi}\frac{\partial E}{\partial f_{AC}}. \tag{3}$$

On the other hand, a particle with magnetic moment $\mu \equiv \sigma_z \mu_B \hat{z}$ can be viewed as a flux line with the flux $\Phi_\mu = 4\pi\mu/l$. Because of this, an electric field ε is essentially induced when this particle moves with velocity v. According to Faraday's law:

$$\nabla \times \varepsilon = -\frac{1}{c}\frac{\partial B_\mu}{\partial t}$$

we can obtain $\varepsilon = \frac{1}{c} B_\mu \times v$ with $B_\mu = \Phi_\mu / 2\pi Ra$ as the flux density. Consequently, the total electric field generated by the spin currents in the ring can be easily obtained

$$\mathcal{E} = \sum_{n\sigma_z} \varepsilon_{n\sigma_z} = \frac{2}{Ralc}\sum_{n\sigma_z} \mu \times v_n = -\frac{8\pi\mu_B}{la\hbar c}\sum_{n\sigma_z} I_{n\sigma_z}\hat{r} = -\frac{8\pi\mu_B}{la\hbar c} I_s \hat{r} \tag{4}$$

where $\hat{r}$ is the unit vector of radial component. Correspondingly, the induced AC flux is found to be

$$f_{AC} = \frac{\mu_B \mathcal{E}_r R}{\hbar c} = -\frac{8\pi R \mu_B^2}{la\hbar^2 c^2} I_s\,. \tag{5}$$

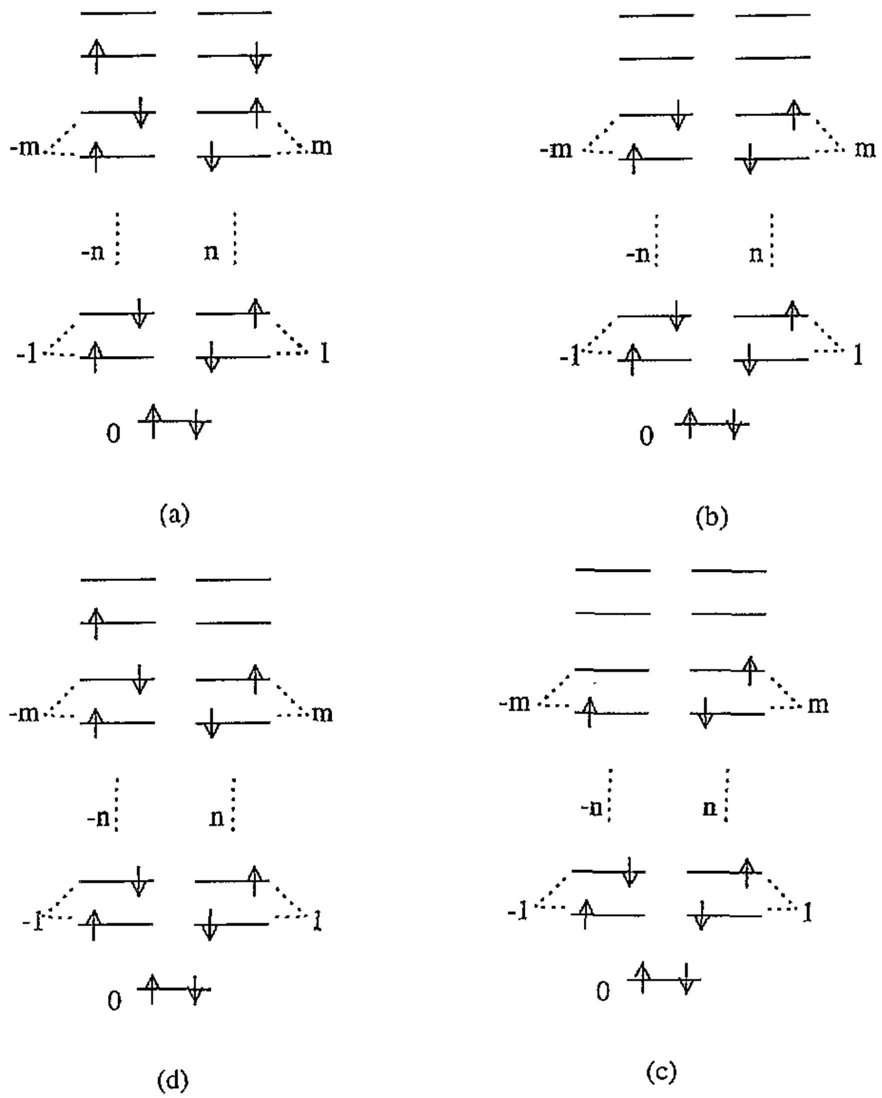

Figure 1. The occupation of electrons for different numbers of electrons in the absence of the magnetic field: (a) $N_e = 4m+4$; (b) $N_e = 4m+2$; (c) $N_e = 4m+1$; (d) $N_e = 4m+3$. The energy level indices of the left and the right columns are $-n$ and n. Note that each level could be split due to the possible AC flux.

At this stage, let us define the quantity describing the degree of spin polarization as $N_p = \sum \sigma_z$. Obviously, in the absence of the magnetic field, the small value of N_p is beneficial to lower the energy of the system. In the following, as an example, we shall discuss in detail the spontaneous AC effect in the case where the number of electrons hosted by the ring is $N_e = 4m+4$ ($m = 0, 1, 2, \ldots$). The schematic drawing for the occupation of electrons with $N_p = 0$ is shown in figure 1(a). Note that the total energy of the system consists of two parts: the energy of particles in the ring E and the energy of the electric field $E_\mathcal{E}$ i.e. $E_\mathrm{T} = E + E_\mathcal{E}$ with

$$E = \frac{\hbar^2}{2m_\mathrm{e}R^2}\left(4\sum_{n=1}^{m} n^2 + 2(m+1)^2 + N_\mathrm{e} f_\mathrm{AC}^2 - N_\mathrm{e} f_\mathrm{AC}\right) \tag{6}$$

and

$$E_\mathcal{E} = \frac{1}{8\pi}\int \mathrm{d}^3x\mathcal{E}^2 = \frac{la\hbar^2c^2}{4R\mu_\mathrm{B}^2} f_\mathrm{AC}^2 \tag{7}$$

where f_AC is chosen to be positive. It is straightforward to find that the total energy E_T reaches its minimum at $f_\mathrm{AC} = 1/2\alpha$ with $\alpha = 1 + alm_\mathrm{e}Rc^2/2N_\mathrm{e}\mu_\mathrm{B}^2$. This is an interesting result, which implies that the AC flux can be spontaneously introduced to lower the energy even though the system is not polarized ($N_\mathrm{p} = 0$). This fact can be understood as follows: the two electrons occupying the highest level have the same contribution to spin currents because one of them with up-spin moves clockwise (negative n) while the other with down-spin moves anti-clockwise (positive n). Moreover, for $N_\mathrm{p} \neq 0$, to make the AC flux state a stable ground state, an external local magnetic field $\boldsymbol{H} = H\hat{\boldsymbol{z}}$ which is applied on the arm of the ring, must be introduced to keep the part of spins aligned. In the presence of this magnetic field, some electrons are forced to jump to higher energy levels. For a given N_p, we can derive the total energy of the system $E_\mathrm{T} = E + E_\mathcal{E} + E_H$ with

$$E_H = \frac{1}{8\pi}\int \mathrm{d}^3xH^2 = laRH^2/4$$

as the energy of magnetic field and the energy of particles in the ring as

$$E(N_\mathrm{p}, H) = \frac{\hbar^2}{2m_\mathrm{e}R^2}\left(2\sum_{n=1}^{m+(N_p+2)/4} n^2 + 2\sum_{n=1}^{m-(N_\mathrm{p}-2)/4} n^2 + N_\mathrm{e} f_\mathrm{AC}^2\right) - N_\mathrm{p}H\mu_\mathrm{B} \tag{8}$$

for $N_\mathrm{p} = 2, 6, \ldots$;

$$E(N_\mathrm{p}, H) = \frac{\hbar^2}{2m_\mathrm{e}R^2}\left(2\sum_{n=1}^{m+N_p/4} n^2 + 2\sum_{n=1}^{m-N_\mathrm{p}/4} n^2 + 2(m+1)^2 + N_\mathrm{e} f_\mathrm{AC}^2 - N_\mathrm{e} f_\mathrm{AC}\right) - N_\mathrm{p}H\mu_\mathrm{B} \tag{9}$$

for $N_\mathrm{p} = 4, 8, \ldots$. From equations (8) and (9), we find directly that the AC flux can be introduced only for $N_\mathrm{p} = 4, 8, \ldots$, and the corresponding total energy $E_\mathrm{T}(N_p, H)$ still reaches its minimum at $f_\mathrm{AC} = 1/2\alpha$. It is also interesting to note that, if the width a and/or the thickness l of the ring approach zero ($\alpha \to 1$), the AC flux approaches a finite constant $\frac{1}{2}$ whereas the spin current $I_s = -la\hbar^2c^2/(16\pi R\mu_\mathrm{B}^2\alpha)$ approaches zero. In this limiting case, the energy of the electric and magnetic field (EM) vanishes and the total energy of the system equals the energy of particles in the ring. Hereafter, for simplicity, we neglect the EM energy in the total energy because of the small width a and thickness l of the ring. As we mentioned before, the spin of a fermion system cannot be spontaneously polarized, so that the condition $E_\mathrm{T}(N_\mathrm{p}, H) \leqslant E_\mathrm{T}(N_\mathrm{p} = 0, H = 0)$ should be satisfied for the stable AC flux state. With this condition, we obtain the threshold value of the magnetic field as $H_\mathrm{c} = N_\mathrm{e}N_\mathrm{p}/16$ (in units of $\hbar^2/2m_\mathrm{e}R^2\mu_\mathrm{B}$). If some typical values $R \sim 10^2$ nm and $N_\mathrm{e} \sim 10^2$ are taken, the order of the magnitude of H_c is estimated to be $10^3 N_\mathrm{p}$ Gauss. At present, it is clear that the magnitude of the magnetic field needed for observable spontaneous AC flux in partially spin-polarized states, particularly with relatively small N_p, is within the achievable region for current equipment; while the fully spin-polarized state is difficult to be realized.

In a similar way, we are able to investigate the spontaneous AC effect in a partially spin-polarized ring which hosts other numbers of electrons. In the following, we directly write down the AC flux, which can be spontaneously introduced to lower the energy of the system for a specific set of N_p, and the threshold value of the magnetic field to keep the flux state stable.

(i) When $N_e = 4m + 2$ ($m = 0, 1, \ldots$), figure 1(b),

$$f_{AC} = \frac{1}{2\alpha} \qquad H_c(N_p) = \frac{N_e N_p}{16} \tag{10}$$

where $N_p = 2, 6, \ldots, N_e$.

(ii) When $N_e = 4m + 1$ ($m = 1, 2, \ldots$), figure 1(c),

$$f_{AC} = \frac{N_e \pm N_p}{4N_e\alpha} \tag{11}$$
$$H_c(N_p) = \frac{N_e(\pm N_p + 1)(\pm N_p N_e - N_e + 2) + (N_e - 1)^2}{16N_e(N_p - 1)} + \frac{2N_e f_{AC}^2 - (N_e \pm N_p) f_{AC}}{2(N_p - 1)}$$

where $N_p = 3, 7, \ldots, N_e - 2$ for '+' while $N_p = 5, 9, \ldots, N_e$ for '−'.

(iii) When $N_e = 4m + 3$ ($m = 0, 1, \ldots$), figure 1(d),

$$f_{AC} = \frac{N_e \pm N_p}{4N_e\alpha} \tag{12}$$
$$H_c(N_p) = \frac{N_e(\pm N_p - 1)(\pm N_p N_e + N_e + 2) + (N_e + 1)^2}{16N_e(N_p - 1)} + \frac{2N_e f_{AC}^2 - (N_e \pm N_p) f_{AC}}{2(N_p - 1)}$$

where $N_p = 5, 9, \ldots, N_e - 2$ for '+' while $N_p = 3, 7, \ldots, N_e$ for '−'.

Now we can conclude that: (a) The result for $N_e = 4m + 2$ is similar to that for $N_e = 4m + 4$. However, when the system is not polarized, the AC flux cannot be spontaneously introduced, which is quite different from the case when $N_p = 0$ for $N_e = 4m + 4$. (b) When $N_e = 4m + 1$, the spontaneous AC flux increases with N_p ($= 3, 7, \ldots, N_e - 2$) from $(N_e + 3)/4N_e\alpha$ to $(N_e - 1)/2N_e\alpha$, while it decreases with N_p ($= 5, 9, \ldots, N_e$) from $(N_e - 5)/4N_e\alpha$ to zero. The threshold value of the magnetic field H_c is of the order of $N_e N_p/16$ for $N_e,\ N_p \gg 1$. (c) For $N_e = 4m + 3$, the spontaneous AC flux increases with N_p ($= 5, 9, \ldots, N_e - 2$) from $(N_e + 5)/4N_e\alpha$ to $(N_e - 1)/2N_e\alpha$ while it decreases with N_p ($= 3, 7, \ldots, N_e$) from $(N_e - 3)/4N_e\alpha$ to zero. The order of the magnitude of H_c is also about $N_e N_p/16$. All of these results provide us with important information that the possibly observable AC effect ($f_{AC} \sim 1/4\alpha$–$1/2\alpha$) could exist in most cases regardless of the number of electrons in the system.

Finally, we wish to point out that the pure AC effect investigated previously in fully spin-polarized states [4] is merely a specific and unusual case. The present results demonstrate clearly that the spontaneous AC flux in partially spin-polarized states could more easily be tested in some well-designed experiments, particularly for some neutral fermion systems.

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